Find a reduction formula for $F_n(x) = \int \sin^n x \, dx$.

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1/9/25

$$F_n(x) = \int \sin^{n-2} x \,(1 - \cos^2 x)\, dx$$

$$= \int \sin^{n-2} x \, dx - \int \sin^{n-2} x \cos^2 x \, dx$$

$$\int \sin^{n-2} x \cos x \, dx$$

$$= \int u^{n-2} \, du$$

$$= \frac{u^{n-1}}{n-1} + C$$

$$= \frac{\sin^{n-1} x}{n-1} + C$$

$$\int \sin^{n-2} x \cos^2 x \, dx$$

$$= \int \sin^{n-2} x \cos x \cdot \cos x \, dx$$

$$= \cos x \int \sin^{n-2} x \cos x \, dx - \int (-\sin x) \int \sin^{n-2} x \cos x \, dx$$

$$= \cos x \cdot \frac{\sin^{n-1} x}{n-1} + \int \sin x \cdot \frac{\sin^{n-1} x}{n-1} + C \sin x \, dx$$

$$= \frac{1}{n-1} \left(\sin^{n-1} x \cos x + \int \sin^n x \, dx + \int C \sin x \, dx \right)$$

$$= \frac{1}{n-1} \left(\sin^{n-1} x \cos x + F_n(x) - C \cos x \right)$$

$$F_n(x) = F_{n-2}(x) - \frac{1}{n-1} \left(\sin^{n-1} x \cos x + F_n(x) - C \cos x \right)$$

$$\Rightarrow \left(1 + \frac{1}{n-1}\right) F_n(x) = F_{n-2}(x) - \frac{1}{n-1} \left(\sin^{n-1} x \cos x - C \cos x \right)$$

$$\therefore F_n(x) = \frac{n-1}{n} F_{n-2}(x) - \frac{1}{n} \left(\sin^{n-1} x \cos x - C \cos x \right)$$